MATHEMATICS 3C/3D

**Section Two:
Calculator-assumed**

Please place your student identification label in this box

Student Number: In figures

In words _____

_____

Number of additional
answer booklets used
(if applicable):

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                    correction tape/fluid, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
                    and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

MAT3CD-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed   **(100 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

Question 9   **(4 marks)**

Let $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x+5}$.

(a)   Determine an expression for $f(g(x))$.   (1 mark)

(b)   Determine the domain of $f(g(x))$.   (1 mark)

(c)   For what value(s) of x will $f(f(x)) = x$?   (2 marks)

Question 10 **(11 marks)**

A company makes two models of aircraft, the Airglide and the Skymaster.

The Airglide requires 200 hours of labour and costs $100 000 to make. The Skymaster requires 100 hours of labour and costs $200 000 to make.

Each month, the company can spend at most $1 200 000, and can use up to 1200 hours of labour. It needs to make at least seven aircraft each month, but no more than ten.
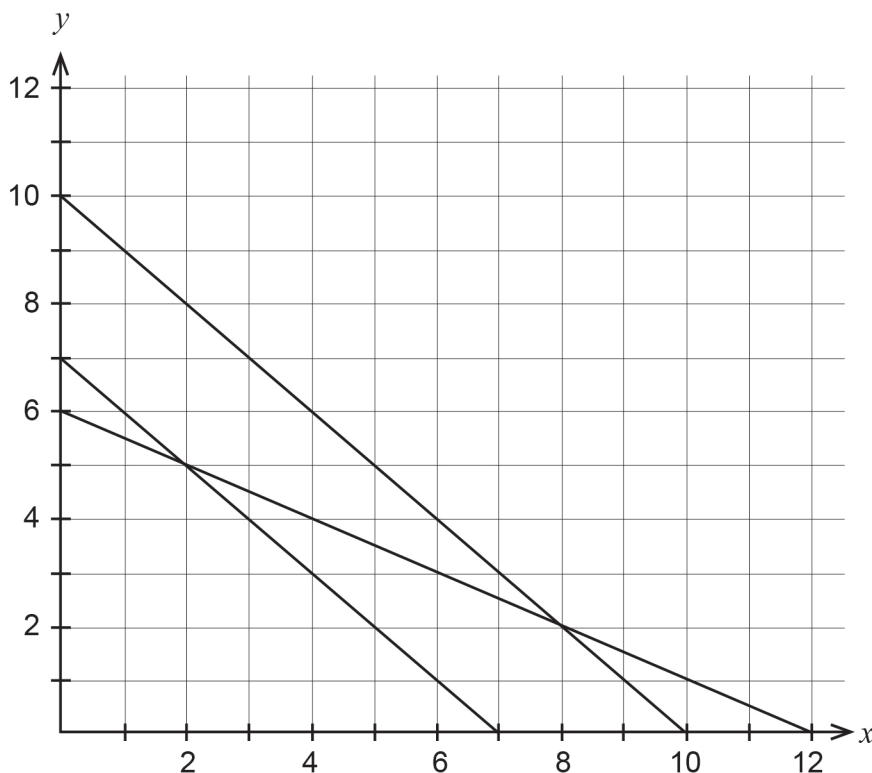
Let x = the number of Airglide aircraft produced each month, and
        y = the number of Skymaster aircraft produced each month.

Some of the constraints relating to the information above can be represented by the following inequalities:

$$x \geq 0 \quad y \geq 0 \quad x + y \geq 7 \quad x + y \leq 10 \quad x + 2y \leq 12.$$

(a)   State one more inequality which, along with those shown above, is sufficient to determine the feasible region. (1 mark)

(b)   Draw this inequality on the axes below and shade the feasible region. (2 marks)

(c)   Each Airglide aircraft produced makes a profit of $500 000 for the company and each Skymaster aircraft makes a profit of $300 000.

Determine the number of each model that should be made each month in order to maximise the total profit, and state this maximum profit.   (4 marks)

(d)   By how much can the profit on each Airglide aircraft be reduced before the optimal number of aircraft in Part (c) above is changed?   (4 marks)

Question 11 **(5 marks)**

Iodine-131 is present in radioactive waste from the nuclear power industry.

It has a half-life of eight days. This means that every eight days, one half of the iodine-131 decays to a form that is not radioactive.

This decay can be represented by the equation $N = N_0 e^{kt}$,

where N = amount of iodine-131 present after t days, and
N_0 = amount of iodine-131 present initially.

(a) Determine the value of k correct to **three (3)** decimal places. (3 marks)

(b) If 125 milligrams of iodine-131 are considered to be safe, how many days will it take for 88 grams of iodine-131 to decay to a safe amount? (2 marks)

See next page

Question 12 **(8 marks)**

The standard deviation of the lifetimes of Xact computer chips is 550 hours.

Quality control experts plan to estimate μ, the mean lifetime of Xact chips, using the mean lifetime of a random sample of Xact chips.

(a) The experts would like to be 95% confident that the mean lifetime of chips in the sample is within 10 hours of μ. How large a sample should they take? (3 marks)

(b) Suppose that a random sample of 100 Xact chips is taken, and the mean lifetime of these chips is 9937 hours.

Based on this sample, determine a 90% confidence interval for μ. (3 marks)

(c) The manufacturer claims that the mean lifetime of Xact chips is at least 10 000 hours. Does the sample in Part (b) provide a strong reason to doubt this claim? Justify your answer. (2 marks)

Question 13 **(8 marks)**

(a)   Ten per cent of a large population is left-handed.

If six people are selected randomly from the population, what is the probability that two or more of these six people are left-handed? (3 marks)

(b)   In a group of 30 people, three are left-handed.

Six people are selected randomly from this group. What is the probability that

(i)   two or more of the six people are left-handed? (3 marks)

(ii)   three of the six people are left-handed, given that two or more are left-handed? (2 marks)

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Question 14 **(10 marks)**

The velocity of a robotic engine moving on a monorail is given by

$v = 3t^2 - 12t + 9$ metres per second, where t = time in seconds.

Determine

(a) the acceleration after 4 seconds. (2 marks)

(b) how far the engine is from its starting point after 10 seconds. (3 marks)

(c) the total distance travelled by the engine in the first 10 seconds. (3 marks)

(d) the average velocity of the engine during the first 10 seconds. (2 marks)
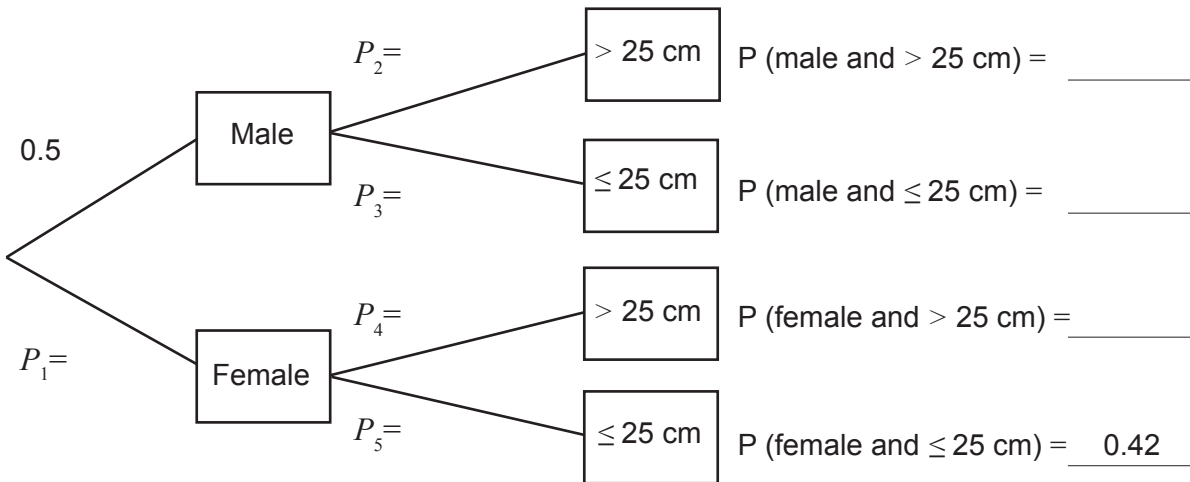
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Question 15 **(9 marks)**

In a population of fish, 50% are male and 50% are female. Overall, 30% of the fish are over 25 cm in length. Furthermore, 42% of the fish are female and 25 cm or under in length.

(a) Use this information to complete the tree diagram below by determining the probabilities P_1 to P_5 on the branches of the diagram, and the probabilities to the right of the tree diagram. **(5 marks)**

$P_2 =$ > 25 cm   P (male and > 25 cm) = _____

   Male

0.5

$P_3 =$ ≤ 25 cm   P (male and ≤ 25 cm) = _____

$P_4 =$ > 25 cm   P (female and > 25 cm) = _____

$P_1 =$ Female

$P_5 =$ ≤ 25 cm   P (female and ≤ 25 cm) = ___0.42___

See next page

(b) What is the probability that a randomly caught fish will be:

    (i) 25 cm or under in length?   (1 mark)

    (ii) either female or over 25 cm in length?   (1 mark)

    (iii) female, given that it is over 25 cm in length?   (2 marks)

Question 16 **(8 marks)**

A spherical balloon has volume $V = \dfrac{4\pi r^3}{3}$, where r is its radius.

(a) Determine an expression for $\dfrac{dV}{dr}$. (1 mark)

(b) The balloon is being inflated at a rate of 100 cm³ per second. At what rate is the balloon's radius increasing at the time when the radius is 5 cm? (3 marks)

(c) Using the formula $\delta V \approx \dfrac{dV}{dr}\,\delta r$, find the approximate percentage increase in the balloon's volume when its radius increases by 1%. (4 marks)

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Question 17 **(6 marks)**

A coffee machine is intended to produce cups of coffee with a mean temperature between 74 °C and 78 °C.

However, the temperature of coffee produced is in fact uniformly distributed between 70 °C and 80 °C, with a mean of 75 °C and a standard deviation of 2.89 °C.

(a) What is the probability that a cup of coffee produced by the machine will have a temperature between 74 °C and 78 °C? (1 mark)

(b) If two cups of coffee are produced by the machine, what is the probability that exactly one of the two cups has a temperature between 74 °C and 78 °C? (2 marks)

(c) Use the Central Limit Theorem to estimate the probability that the mean temperature of the next 50 cups of coffee produced by the machine will lie between 74 °C and 78 °C. Give the answer correct to **two (2)** decimal places. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 18   **(9 marks)**

A new treatment for back pain is being tested.

A trial group consists of 100 randomly chosen patients with back pain. There is a 25% chance that any one of these patients will report an improvement after one month if no treatment is given.

Let X denote the number of patients who will report an improvement after one month, assuming that no treatment is given.

(a)   Is the random variable X discrete or continuous?   (1 mark)

(b)   State the probability distribution of X.   (2 marks)

(c)   Calculate the mean and standard deviation of X.   (2 marks)

(d)   What is the probability that 35 or more of the patients in the trial group will report
        an improvement after one month, assuming that no treatment is given?   (2 marks)

(e)   Now suppose that each patient in the trial group is given the new treatment and that 35
        of them report an improvement after one month. Is this strong evidence that the
        treatment is effective? Justify your answer.   (2 marks)

Question 19   **(7 marks)**

A square sheet of metal has sides of fixed length L cm.
A tray is constructed by cutting smaller square pieces out of the corners of the metal sheet and folding up the sides. Each of the pieces has side length xL cm.
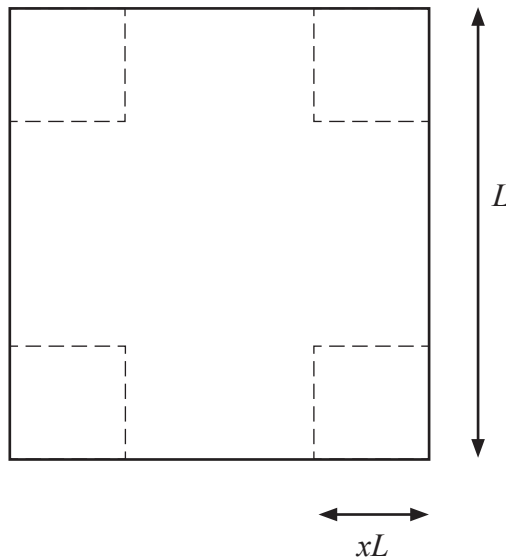
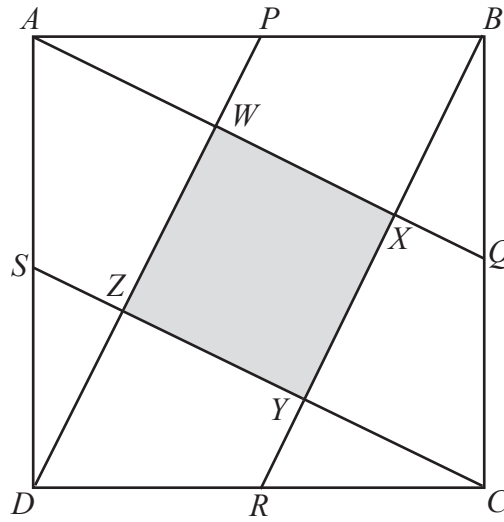(a)   Show that the volume of the tray is given by $V = L^3(x - 4x^2 + 4x^3)$ cm³.   (3 marks)

(b)   What is the maximum possible volume of the tray, in terms of L?   (4 marks)

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In the diagram below, P, Q, R and S bisect the sides of the square $ABCD$.

(a)   Prove that $\triangle AWP$ is similar to $\triangle ABQ$. (3 marks)

See next page

(b) If $ABCD$ has side length 10 cm,

(i) determine the ratio $\dfrac{AQ}{AP}$. (2 marks)

(ii) determine the area of the shaded region $WXYZ$. (3 marks)

Question 21 **(7 marks)**

Let $f(n) = n^2 + n + 11$.

Here are two possible conjectures:

Conjecture P: For each positive integer n, $f(n)$ is prime.
Conjecture Q: For each positive integer n, $f(n)$ is not a multiple of 3.

One of these conjectures is false, and the other is true.

(a)   Which conjecture is false? Justify your answer.   (3 marks)

(b)   Prove the true conjecture.   (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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